Supercurrent multiplet and anomalies

1984 J. Phys. A: Math. Gen. 17 L725

(http://iopscience.iop.org/0305-4470/17/13/009)

J. Phys. A: Math. Gen. 17 (1984), L725–L729. Printed in Great Britain

LETTER TO THE EDITOR

Supercurrent multiplet and anomalies

A Smailagić

Physics Department, Laval University, Québec, Canada, G1K 7P4

Received 9 May 1984

Abstract. We investigate the effect of the surface term of the linearly divergent diagrams on the supercurrent anomaly. It is found that such a term can not be used to shift the anomaly in the conformal part only. As a consequence the multiplet structure of currents seems to be violated.

The issue of anomalies in the framework of supersymmetric theories has been an interesting question since the early work by Ferrara and Zumino (1975). They have shown that the currents of supersymmetric theory and their anomalies lie within corresponding supersymmetric multiplets in the case of the massive Wess–Zumino model. It has been argued later that it is true for all anomalies (Lang 1978), at least in global supersymmetry. The first doubts on this question were raised by the work of Abbott *et al* (1977) who explicitly calculated the supercurrent anomaly for $N=1$ Yang–Mills theory using the Adler (1969) and Rosenberg (1963) method and argued that the coefficient of the anomaly was not consistent with multiplet structure (at the one loop level).

This was shown to be an incorrect statement by explicit calculation (Lang 1978), but can easily be confirmed from multiplet structure at any order using a known form of the trace (Chanowitz and Ellis 1973, Collins *et al* 1967) (or chiral) anomaly. Explicitly, if B is the coefficient of the trace anomaly†, multiplet structure is satisfied with $4B$ being the coefficient of the supercurrent anomaly and in the case of adjoint SU(2) representation it gives $3g^2/4\pi^2$ which is the correct coefficient of Abbott *et al.*

However, the purpose of this letter is not to prove the above, but to make some comments on the multiplet structure of currents and anomalies that has escaped attention and will be complementary to some statements in Abbott *et al* (1977). The discussion is relevant to all later work on this subject since they essentially agree with it.

Let us briefly review the known results relevant to this paper. It has been established that the supercurrent of $N=1$ supersymmetric Yang–Mills theory has an anomaly which forms a multiplet together with the chiral and trace anomalies. Enforcing gauge invariance, the supercurrent anomaly was found in the Poincaré part of the superconformal group ($\partial^\mu J_\mu \neq 0$) by Abbott *et al* (1977), confirmed by the point-splitting technique (Inagaki 1978) and the Siegel dimensional reduction (RDR) scheme (Majmudar *et al* 1980) and contradicted by Curtright (1977) who found an anomaly in the conformal part of the superconformal group ($\gamma J \neq 0$).

† $B \equiv \beta(g)/2g = -(3C(G)/32\pi^2)g^2$ (one loop); $C=2$ for $I=1$ SU(2) multiplet.

0305-4470/84/130725 +05$02.25

It has been stated that the two forms of anomaly are not incompatible since they can be transformed into each other by the routing of momenta in Feynman graphs, recalling the familiar situation in the chiral case.

We would like to show in this letter that this hope is not fulfilled i.e. it is not a matter of simply redefining momenta that would shift the anomaly from $\partial^\mu J_\mu \neq 0$ into $\gamma J \neq 0$. This will have important implications on the multiplet structure of currents and anomalies.

The model is $N=1$ supersymmetric Yang–Mills theory described by a vector multiplet in the adjoint representation of SU(2) whose Lagrangian is

$$\mathscr{L} = -\tfrac{1}{4}F^a_{\alpha\beta}F^{\alpha\beta a} + \tfrac{1}{2}\mathrm{i}\bar{\psi}^a \not{D}\psi^a \tag{1}$$

with the action invariant under supersymmetry transformations

$$\delta A^a_\mu = \mathrm{i}\bar{\alpha}\gamma_\mu\psi^a \tag{2}$$

$$\delta\psi^a = F^a_{\alpha\beta}\sigma^{\alpha\beta}\alpha. \tag{3}$$

The theory is invariant under conformal supersymmetry described by charges Q (Poincaré part) and S (conformal part) defined as space integrals of the time component of supercurrents

$$J_\mu = \mathrm{i}F^a_{\alpha\beta}\sigma^{\alpha\beta}\psi^a \tag{4}$$

$$K_\mu = -\mathrm{i}\gamma x J_\mu. \tag{5}$$

The conditions of conformal supersymmetry are

$$\partial^\mu J_\mu = 0 \tag{6}$$

$$\gamma^\mu J_\mu = 0. \tag{7}$$

At the quantum level the supercurrent has an anomaly (Abbott *et al* 1977)†

$$\partial^\mu J_\mu = -(3g^2/4\pi^2)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\gamma^\mu\partial_\nu\psi). \tag{8}$$

However, in calculating this anomaly by the Adler–Rosenberg method the contribution from the surface terms has been omitted as being ambiguous. Following the Gross *et al* (1972) prescription for chiral anomaly it is these terms that are relevant for the shift of anomaly from one part to the other (i.e. rerouting the momenta and without such terms in supersymmetric cases we are left with the hope that the same is true for the supercurrent anomaly). In order to evaluate the surface term let us define the full amplitude of the process $\bar{J}_\mu \to A_\mu\psi$ as

$$T_{\mu(p,q|a)} = T_{\mu(p,q)} + S_{\mu(p,q|a)} \tag{9}$$

where S_μ denotes the surface term and a is the arbitrary routing of momenta. The diagrams that are formally linearly divergent are given in figure 1.

It is interesting to notice that the third diagram gives no surface term. Independently of the γ matrix structure we can evaluate the surface term to be

$$S^{\alpha\rho\omega}_\mu = \tfrac{1}{12}\pi^2(a_\alpha\delta_{\rho\omega} + a_\rho\delta_{\alpha\omega} + a_\omega\delta_{\alpha\rho}). \tag{10}$$

The full surface term, taking into account the γ matrix structure of each diagram and

† We would like to mention that from now on the identity $\partial^\mu J = 0$ is to be understood as an on-shell condition for matrix elements.

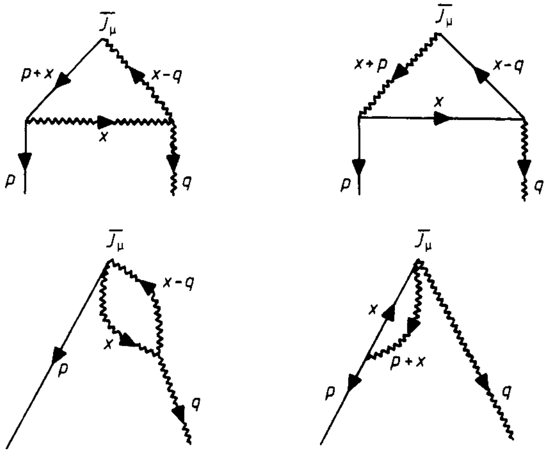

Figure 1.

summing them up, can be found to be†

$$S_\mu = C[\not{a}(\delta\mu\rho + 18\sigma_{\mu\rho}) + 13(a_\rho\gamma_\mu - 2a_\mu\gamma_\rho)]\bar{u}_{(p)}e_{\rho(q)}. \tag{11}$$

Now, shifting the anomaly from one side to the other corresponds to finding such an a (p, q) that would kill the anomaly in one part i.e. $(\partial^\mu J_\mu = 0)$ and restore it in the other $(\gamma J \neq 0)$. To show that it is *not* possible we merely notice that

$$\gamma^\mu S_{\mu(p,q|a)} = 0 \tag{12}$$

for *any* choice of a. This simply means that this condition remains independent of the routing and with any choice can not restore $\gamma J \neq 0$. This is essentially due to the fact that it is an algebraic constraint and depends only on the γ matrix structure of the theory which is unchanged by a particular routing. There is no way to evaluate the anomaly in the γJ part. In fact, the only result that has done so (Curtright 1977) was obtained using regular dimensional regularisation which is known to be highly unsuitable for supersymmetric theories (Copper *et al* 1980) as it provides no supersymmetry preservation already at one loop level. Besides, a sample calculation was done for the Wess–Zumino model which has a completely different nature of anomalies.

The results indicate that a regularisation scheme that would give the anomaly in conformal part, while preserving Poincaré supersymmetry hardly exists.

However, $\gamma J = 0$ *does not* mean that we preserve conformal supersymmetry either, since $\partial^\mu K_\mu = -\mathrm{i}\gamma x \partial^\mu J_\mu \neq 0$ (even if $\gamma J = 0$), rather that conformal supersymmetry is broken in full.

To confirm the result let us look at the gauge and Poincaré invariance conditions. We choose $a_\mu = \alpha p_\mu + \beta q_\mu$ as the most general forms in terms of external momenta. The gauge condition is

$$q^\mu T_\mu = \text{constant } \alpha(qp_\mu - 2pq\gamma_\mu)\bar{u}(p)e_\mu(q) \tag{13}$$

and Poincaré supersymmetry

$$(p+q)^\mu T_\mu = \text{constant } [(13\alpha - 8\beta)qp_\mu - (26\alpha + 8\beta)pq\gamma_\mu]\bar{u}_{(p)}e_{\mu(p)}. \tag{14}$$

† The constant that appears will not be needed, but for the sake of detail it is $C = (\pi^2 g^2/144)K$; where K is the casimir operator of adjoint representation of SU(2).

By explicit calculation (Majmudar *et al* 1980) we know that with the choice of momenta as in figure 1, gauge invariance is maintained $(q^\mu T_\mu(p,q)=0)$ so we are forced to choose $\alpha=0$. With such a choice $(p+q)^\mu S$ is even in *form* different from (8) and can not be used to cancel the anomaly so we have to choose $\beta=0$. With the choice $\alpha=0$, $\beta=0$ gauge invariance is maintained, but there is an anomaly in $\partial^\mu J_\mu \neq 0$. From (10) we see that, in principle, there is a choice of a that could kill the anomaly (i.e. there is a general solution to (10) with α, $\beta \neq 0$ which gives the same form of the surface term as the anomaly), but we would lose gauge invariance.

A similar result was found by Abbott *et al* (1977), but it has not been realised that the clash is between gauge invariance and Poincaré supersymmetry invariance, without disturbing $\gamma J=0$. The attempt to preserve all three has no justification.

Therefore we could conclude that the explicit calculation, always indicates the presence of an anomaly in $\partial^\mu J_\mu=0$ and leads to breakdown of both Poincaré and conformal supersymmetry and routing of momenta *can not* establish an anomaly only in the conformal part of supersymmetric group. This is a completely new result compared with the chiral case where such a shift is possible.

On the other hand, we tend to believe that quantum corrections should not disturb Poincaré supersymmetry (Wess and Zumino 1974). The only way out would be to redefine the supercurrent as

$$J_\mu = J_\mu^0 + cF^{*a}_{\mu\nu}\gamma^\nu\gamma^5\psi^a \tag{15}$$

that could restore the right order of supersymmetry breaking (on shell)

$$\partial^\mu J_\mu = 0 \tag{16}$$

$$\gamma^\mu J_\mu = 2cF^{*a}_{\alpha\beta}\sigma^{\alpha\beta}\psi^a. \tag{17}$$

We would like to argue that even this is not the way out. The reason is the following. With redefinition (15) we have restored the multiplet structure of anomalies (which was broken by $\gamma J=0$), but we have to preserve at the same time the structure of multiplet of currents. Any redefinition of any of the currents would immediately lead to the redefinition of the others *if* the multiplet of currents is to be preserved. This is not bad as long as such redefinition does not violate (12) (as it happens in the case of the Wess–Zumino model (Ferrara and Zumino 1975). Direct variations of (15) do not satisfy the structure of the multiplet of currents, and we were not able to find any consistent way of redefining the chiral current, and preserving the energy–momentum tensor (16) and (17) at the same time. This would merely indicate that formal redefinition (15), although giving (17), does not define the supersymmetric generator Q, and thus we are back to (8).

Knowing that the multiplet of currents is not unique, once conformal symmetry is broken, we tried to use a new minimal multiplet of West (1981), by redefining the chiral current as

$$\tilde{j}^{(5)}_\mu = j^{(5)}_\mu - X_\mu \tag{18}$$

$$X_\mu = F^{*a}_{\mu\nu}A^{\nu a} - \tfrac{1}{6}g\varepsilon_{abc}\varepsilon_{\nu\mu\alpha\beta}A^{\nu a}A^{\alpha b}A^{\beta c} \tag{19}$$

$$\partial^\mu \tilde{j}^{(5)}_\mu = 0 \tag{20}$$

at the expense of gauge invariance hoping that this multiplet could lead to meaningful redefinitions, but without success.

We would finally conclude that the explicit calculation of supercurrent anomalies for $N = 1$ Yang–Mills theory indicates a breakdown of both the Poincaré and the conformal part of the supersymmetry group at the quantum level and no way could be found to shift the anomaly in the conformal part only, as desired. In view of the renormalisation schemes that are supposed to preserve (at least global) supersymmetry, Siegel's (1979) method seemed promising, but still gave an anomaly in the Poincaré part of the supersymmetry group (Majmudar *et al* 1980). This can be understood in the light of recent results (Avdeev and Vladimir 1983) showing that such a method fails at high order of perturbation theory, while the multiplet structure of anomalies is supposed to hold at any order, anomalies being proportional to the β function.

We are then only left with the restricted choice of possibly finite theories (at all orders) such as $N = 4$ or $N = 8$ as being anomaly free. The inclusion of matter multiplets is of no help since such a model is anomaly free only at the first order (Jones 1975).

I would like to thank Professor A Salam of the ICTP, Trieste, Italy, where this work has started and Professor Q Ho-Kim of the University Laval where it has been completed.

References

Abbott L T, Grisaru M T and Schintzer H J 1977 *Phys. Rev.* D **16** 2955
Adler S 1969 *Phys. Rev.* **177** 2426
Avdeev L A and Vladimir A A 1983 *Nucl. Phys.* B **219** 262
Chanowitz M S and Ellis J 1973 *Phys. Rev.* D **7** 2490
Collins J C, Duncan A and Jogether S D 1977 *Phys. Rev.* D **16** 438
Copper D M, Jones D R T and van Nieuwenhuizen P 1980 *Nucl. Phys.* B **167** 479
Curtright T 1977 *Phys. Lett.* **71B** 185
Ferrera S and Zumino B 1975 *Nucl. Phys.* B **87** 207
Gross D J, Jackiw R and Treiman S B 1972 *Lectures on current algebra and its applications* (Princeton: Princeton University Press) p 139
Iiopulos J and Zumino B 1974 *Nucl. Phys.* B **76** 310
Inagaki H 1978 *Phys. Lett.* **77B** 56
Jones D R T 1975 *Nucl. Phys.* B **87** 127
Lang W 1978 *Nucl. Phys.* 201
Majmudar P, Poggio E C and Schintzer H 1980 *Phys. Lett.* **93B** 321
Rosenberg L 1963 *Phys. Rev.* **129** 2786
Siegel W 1979 *Phys. Lett.* **84B** 193
Wess J and Zumino B 1974 *Phys. Lett.* **49B** 52
West P C 1984 *Current in supersymmetric theories, Lectures at Moscow Quantum Gravity Conference 1981*